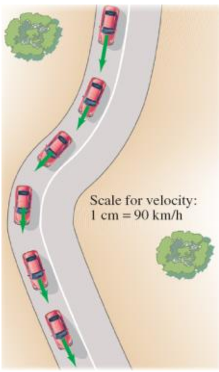
I. Vectors and Scalars



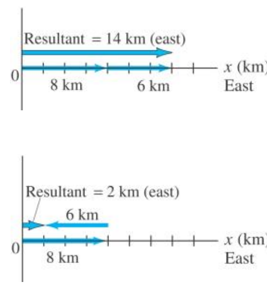
A **vector** has **magnitude** as well as **direction**.

Some vector quantities: **displacement**, **velocity**, **force**, **momentum**

A **scalar** has only a **magnitude**.

Some scalar quantities: **mass**, **time**, **temperature**

II. Addition of Vectors—Graphical Methods



For vectors in one dimension, simple **addition** and **subtraction** are all that is needed.

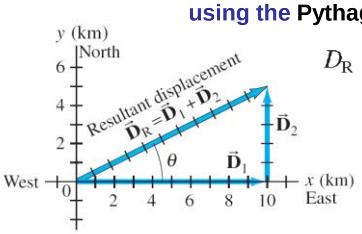
You do need to be careful about the **signs**, as the figure indicates.

a. Addition of Vectors—Graphical Methods

If the motion is in two dimensions, the situation is somewhat more complicated.

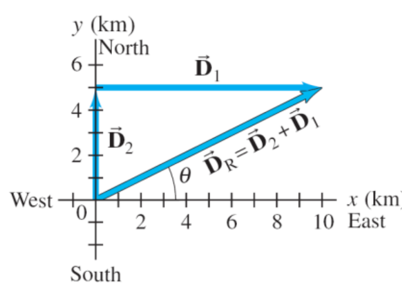
Here, the actual travel paths are at right angles to one another; we can find the **displacement** by using the **Pythagorean Theorem**.

$$D_R = \sqrt{D_1^2 + D_2^2}$$



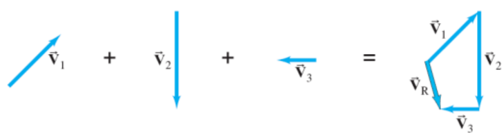
b. Addition of Vectors—Graphical Methods

Adding the vectors in the **opposite** order gives the same result: $\vec{V}_2 - \vec{V}_1 = \vec{V}_2 + (-\vec{V}_1)$.



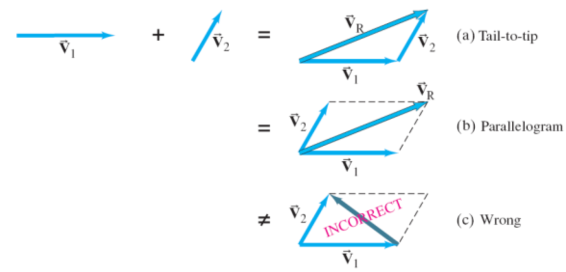
c. Addition of Vectors—Graphical Methods

Even if the vectors are not at right angles, they can be added **graphically** by using the **tail-to-tip** method.

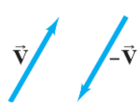


d. Addition of Vectors—Graphical Methods

The **parallelogram** method may also be used; here again the vectors must be tail-to-tip.



e. Subtraction of Vectors, and Multiplication of a Vector by a Scalar

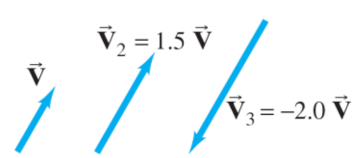


In order to **subtract** vectors, we define the **negative** of a vector, which has the same magnitude but points in the **opposite** direction.

Then we add the negative vector.

III. Subtraction of Vectors, and Multiplication of a Vector by a Scalar

A vector $\vec{V}$ can be multiplied by a **scalar** c; the result is a vector $c\vec{V}$ that has the same **direction** but a **magnitude** cV. If c is **negative**, the resultant vector points in the opposite direction.

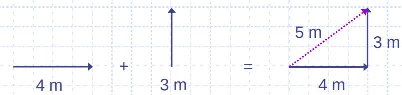


a. Adding & Subtracting Vectors

- Vectors can be added or subtracted from each other graphically.
- Each vector is represented by an arrow with a length that is proportional to the magnitude of the vector.
- Each vector has a direction associated with it.
- When two or more vectors are added or subtracted, the answer is called the **resultant**.
- A resultant that is equal in magnitude and opposite in direction is also known as an **equilibrant**.

b. Adding Vectors using the Pythagorean Theorem

If the vectors occur such that they are perpendicular to one another, the Pythagorean theorem may be used to determine the resultant.



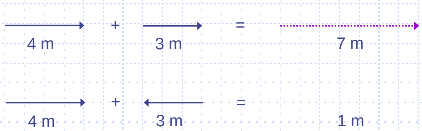
$$A^2 + B^2 = C^2$$

$$(4m)^2 + (3m)^2 = (5m)^2$$

When adding vectors, place the tail of the second vector at the tip of the first vector.

c. Adding & Subtracting Vectors

If the vectors occur in a single dimension, just add or subtract them.

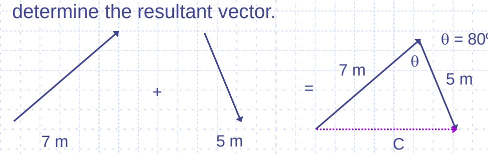


When adding vectors, place the tail of the second vector at the tip of the first vector.

When subtracting vectors, invert the second one before placing its tail at the tip of the first vector.

d. Law of Cosines

If the angle between the two vectors is more or less than 90º, then the Law of Cosines can be used to determine the resultant vector.



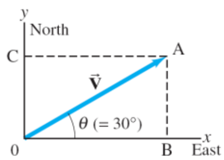
$C^2 = A^2 + B^2 – 2ABCos\ \theta$

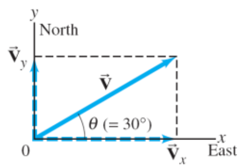
$C^2 = (7m)^2 + (5m)^2 – 2(7m)(5m)Cos\ 80^o$

$C = 7.9$ m

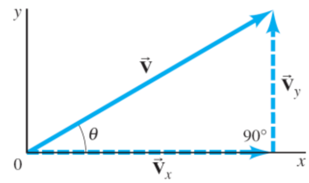
IV. Adding Vectors by Components

Any vector can be expressed as the **sum** of two other vectors, which are called its **components**. Usually the other vectors are chosen so that they are **perpendicular** to each other.





a. Adding Vectors by Components



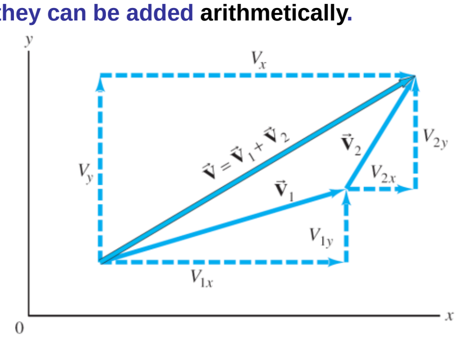
Remember:
soh
cah
toa

$$\sin\theta = \frac{V_y}{V}$$
$$\cos\theta = \frac{V_x}{V}$$
$$\tan\theta = \frac{V_y}{V_x}$$
$$V^2 = V_x^2 + V_y^2$$

If the components are **perpendicular**, they can be found using **trigonometric functions.**

b. Adding Vectors by Components

The **components** are effectively one-dimensional, so they can be added **arithmetically.**



c. Adding Vectors by Components

Adding vectors:

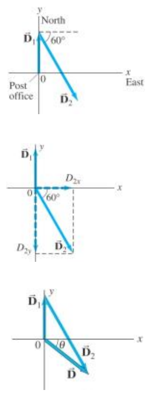
1. Draw a **diagram**; add the vectors **graphically.**
2. **Choose** *x* and *y* axes.
3. **Resolve** each vector into *x* and *y* **components.**
4. **Calculate** each component using sines and cosines.
5. **Add** the components in each direction.
6. To find the **length** and **direction** of the vector, use:

$$V = \sqrt{V_x^2 + V_y^2} \quad \text{and} \quad \tan\theta = \frac{V_y}{V_x}.$$

d. Adding Vectors by Components

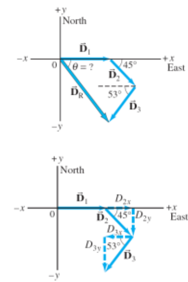
Example 1: Mail carrier's displacement.

A rural mail carrier leaves the post office and drives 22.0 km in a northerly direction. She then drives in a direction 60.0° south of east for 47.0 km. What is her displacement from the post office?



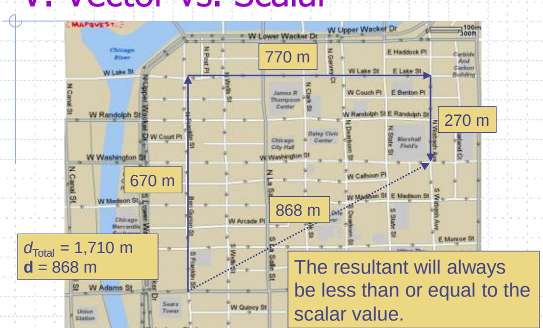
Example 2: Three short trips.

An airplane trip involves three legs, with two stopovers. The first leg is due east for 620 km; the second leg is southeast (45°) for 440 km; and the third leg is at 53° south of west, for 550 km, as shown. What is the plane's total displacement?



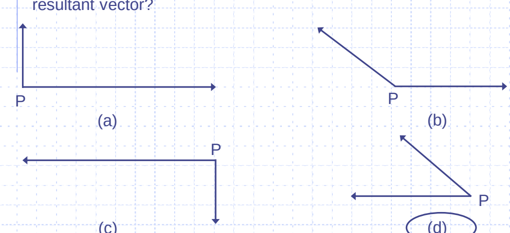
Vector	Components x (km)	Components y (km)
$\vec{D}_1$	620	0
$\vec{D}_2$	311	−311
$\vec{D}_3$	−331	−439
$\vec{D}_R$	600	−750

V. Vector vs. Scalar

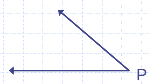


VI. More Examples: Example 3

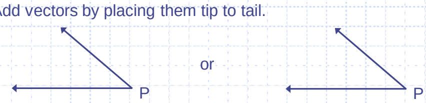
The vector shown to the right represents two forces acting concurrently on an object at point P. Which pair of vectors best represents the resultant vector?



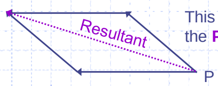
How to Solve:



1. Add vectors by placing them tip to tail.



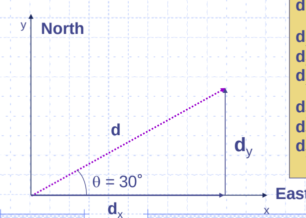
2. Draw the resultant.



This method is also known as the **Parallelogram Method**.

Example 4:

- A bus travels 23 km on a straight road that is 30° North of East. What are the component vectors for its displacement?



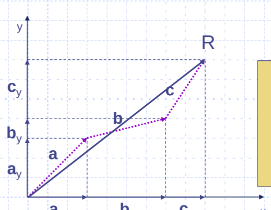
d = 23 km

$\mathbf{d}_x = \mathbf{d} \cos\theta$
$\mathbf{d}_x = (23 \text{ km})(\cos 30°)$
$\mathbf{d}_x = 19.9 \text{ km}$

$\mathbf{d}_y = \mathbf{d} \sin\theta$
$\mathbf{d}_y = (23 \text{ km})(\sin 30°)$
$\mathbf{d}_y = 11.5 \text{ km}$

VII. Algebraic Addition

- In the event that there is more than one vector, the x-components can be added together, as can the y-components to determine the resultant vector.



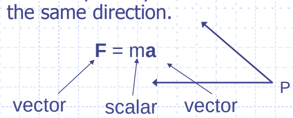
$R_x = a_x + b_x + c_x$

$R_y = a_y + b_y + c_y$

$R = R_x + R_y$

VIII. Properties of Vectors

- A vector can be moved anywhere in a plane as long as the magnitude and direction are not changed.
- Two vectors are equal if they have the same magnitude and direction.
- Vectors are **concurrent** when they act on a point simultaneously.
- A vector multiplied by a scalar will result in a vector with the same direction.

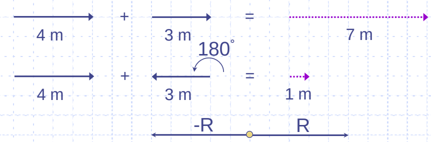


Properties of Vectors (cont.)

- Two or more vectors can be added together to form a **resultant**. The resultant is a single vector that replaces the other vectors.
- The maximum value for a resultant vector occurs when the angle between them is 0°.
- The minimum value for a resultant vector occurs when the angle between the two vectors is 180°.
- The **equilibrant** is a vector with the same magnitude but opposite in direction to the resultant vector.

Properties of Vectors (cont.)

If the vectors occur in a single dimension, just add or subtract them.



IX. Key Ideas

- Vector: Magnitude and Direction
- Scalar: Magnitude only
- When drawing vectors:
  - Scale them for magnitude.
  - Maintain the proper direction.
- Vectors can be analyzed graphically or by using coordinates.